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Lab 9: Oscillations and Differential Equations

AP Physics C Mechanics

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Objective: I can derive a differential equation to describe Newton's Second Law for a spring-mass system by ranking systems with different parameters and explaining the connect between oscillations and Newton's Laws of motion.

To solve for the period of motion for a harmonic oscillator, we can do a force and energy analysis and take into account quantities such as the velocity of the mass at different points in time of the trajectory. This is what you did in AP Physics 1. However, this gets tedious, especially for some of the more complicated examples of harmonic oscillators like the physical pendulum in which you have to use angular momentum and Newton's 2nd Law of rotation to solve for angular speed.

Thankfully, there is a much, much easier method (once you get used to it of course) for finding the period of motion and it helps shed some light on why the motion is described by a sinusoidal function! Please welcome back Differential Equations! After a couple month long hiatus since resistive forces, they are back to help you mathematically solve simple harmonic motion problems.

Today, you will be working with the mass-spring system to conceptually describe the differential equations that we will be using. The differential equations will be differential equations of position for the mass attached to the spring.

In addition, you will also be measuring the period of oscillation for the springs. This will be accomplished by using stopwatches.

Materials

- Motion sensor and PASCO interface
- Springs
- Bronze masses between 5 g and 200 g.
- Ring stand

Procedure

1. Open PASCO Capstone and, if prompted, select "Sensor Data". You need a display (any of them) with a graph. Nothing else is really needed.
2. Change the labels of the axes on your graph to show "acceleration" on the vertical and "position" on the horizontal axes.

Part 1 – Finding spring constant

1. Find the spring constant by hanging a mass from the spring, and measuring its displacement from before the mass was placed on the spring. Use Newton's Second Law (you may need paper!) to calculate the spring constant.

Part 2 – Acceleration vs. Position, Period

1. Pull a 100 g mass down slightly and release it from rest.
2. After a few seconds, begin data collection.
3. What shape does the graph have? It should be linear, and if so, then find the slope of the graph.
4. Record in the data table below. Include at least one of your graphs as a screenshot in this report.
5. Repeat with a different mass (200 g) hanging from the spring.

Part 3 – Period vs. Amplitude

1. For one mass only, time the period of oscillation twice, if you pull the spring down different amounts both times. Record the timing and displacements from equilibrium below.

Displacement 1: 0.27 m Period 1: 1.05 sec.
Displacement 2: 0.51 m Period 2: 0.95 sec

Results and Discussion

Table 1. A list of masses used in the lab, along with data obtained from Capstone and the period.

Spring Constant (N/m, same for all rows)	**Mass (kg)**	**Slope of graph (???)**	**% error once identify what slope should be**
3.16	0.1	-239.58 m/s^2/m	658 (this is not a joke)
3.16	0.2	-43.58 m/s^2/m	37.8

What do you notice about the slope of your graphs (Column 3) in each row, and how they compare to the spring constant and the mass?

The graph slope is supposed to be –spring constant/mass. The actual values recorded seem to vary with changing mass, but could not reasonably be assumed to be directly related to the spring constant and mass.

Noting that your values in Column 3 were a slope from an acceleration vs. position graph, relate the acceleration of the mass with its displacement from equilibrium (aka "position") at any point in time, its actual mass, and the spring constant using an expression.

$$a = -\frac{k}{m}x$$

Post Lab Questions

1. Write out Newton's Second Law for a horizontal Mass-spring system (such that gravity cancels with normal force on the mass) at any general point in its motion if the equilibrium is at x = 0, and replace acceleration with $\ddot{x}$ (the second derivative of position). Does the expression for acceleration match your observations?

$$ma = -kx$$

No, but our observations were incorrect and we ignored gravitational influence so the law is correct. With great percent error comes great explanation. After the mass was released and allowed to freely move on the spring, it not only oscillated in the vertical direction(caused mainly by the force applied to it by the spring). However, the mass also oscillated slightly in the horizontal direction, functioning like a pendulum attached to a spring. This horizontal oscillation was likely due to how the spring was slightly horizontally displaced when we released the mass attached to the spring, so in addition to exerting a restoring force on the mass in the vertical direction, a restoring force would be exerted on the mass in a horizontal direction.
The way we positioned the capstone motion sensor – right below the mass before it was released – prevents it from tracking the mass from slight horizontal displacements. This means that the horizontal oscillation of our mass could have lead to wildly inaccurate data by causing the mass to frequently go out-of-range of the motion sensor, generating inaccurate graphs. Perhaps one or two instances where the mass went out-of-range could be corrected by excluding visual oddities in the graph produced by Capstone. However, the consistent horizontal oscillation of the mass would have lead it to frequently go out of range of the sensor, contaminating the entirety of the data with irregularities. Capstone may have significantly overestimated the acceleration of the mass whenever it went out of range. This would have inflated the acceleration values in relation to the displacement, producing a much larger slope than expected.

Another source of error is that we weren't very experienced using the Capstone software. We may have made a small error in selecting a part of the graph to take a slope from, leading to inaccurate data. Any physical factor would be unlikely to skew our data to the extent that it deviates from expected values, suggesting that the source of the error lies in the way we processed the collected data.

2. What do you notice about the period in your two trials from Part 3?
   It stays about the same for a change in amplitude, suggesting that the period/angular frequency of a spring system does not depend on the amplitude of the oscillation in an ideal situation.